

Effects of the Slot-Harmonics on the Unbalanced Magnetic Pull in an Induction Motor with an Eccentric Rotor

P. Frauman, A. Burakov, A. Arkkio

Abstract—The slotting of electrical rotating machines creates slot-harmonics of the air-gap magnetic flux density. These slot-harmonics are the source of a non-negligible part of the unbalanced magnetic pull (UMP). This paper presents analytical justifications for the use of a numerical impulse method to calculate the effects of the slot-harmonics on the UMP. This method is then used to show that the force components due to the slot-harmonics cannot be reduced easily.

Index Terms—Electrical machines, flux density harmonics, impulse method, rotor eccentricity.

I. INTRODUCTION

WHEN the rotor of an electrical motor is displaced from its central position, a force arises between the rotor and the stator. This force is known as the unbalanced magnetic pull (UMP). The UMP in electrical machines with rotor eccentricity has been known for a long time. However, the recent progresses in numerical calculations have opened new perspectives for the study of this force. It was shown in [1] and [2] that a part of the UMP is due to the first eccentricity harmonics of the air-gap flux density, which appear in all rotating electrical machines with rotor eccentricity. Means to reduce them have been studied, as the use of parallel branches in the stator winding [3]. However, a part of the force has not been explained yet. Nevertheless, some evidences that it might be caused by the slot-harmonics have been presented in [4] and [5]. Also, the attempt to reduce the part of the force due to the slot-harmonics with the use of skewed rotor slots was proved to be unsuccessful in [6].

The purpose of this study is to highlight the effects of the slot-harmonics on the unbalanced magnetic pull of a cage induction motor with an eccentric rotor. This is done numerically using the impulse method in a time stepping finite

elements method (FEM) simulation program. The method used is an extension of the one presented in [2], that was not taking into account the slot-harmonics of the air-gap magnetic flux density. The rotor is moved from its central position for a short period of time during transient finite-element calculations. The magnetic flux density harmonics, including the slot-harmonics, are then calculated from the response and excitation signals. The force acting on the rotor is calculated using the Maxwell stress tensor [7].

The results show that, in general, the slot-harmonics are the source of a non-negligible part of the UMP, and that it appears to be difficult to reduce that part of the force.

II. METHODS OF ANALYSIS

A. Analytical Model for Taking into Account the Slot-Harmonics in the Calculation of the UMP

In an induction motor, the slotting, both on stator and rotor sides, modifies the effective air-gap permeance. A good approximation for the permeance, taking into account the slotting, is given by [8]:

$$\Lambda(\varphi, t) = \frac{\lambda_s(\varphi, t)\lambda_r(\varphi, t)}{\delta} \quad (1)$$

here δ is the radial air-gap length,

$$\lambda_s(\varphi, t) = \frac{1}{k_{C_S}} + \sum_{\nu=1}^{\infty} a_{\nu} \cos(\nu N_S \varphi) \quad (2)$$

is the permeance of the air-gap, without taking into account the rotor slotting effect, and

$$\lambda_r(\varphi, t) = \frac{1}{k_{C_R}} + \sum_{\nu=1}^{\infty} b_{\nu} \cos\left(\nu N_R \varphi - \nu N_R \frac{(1-s)}{p} \omega t\right) \quad (3)$$

is the permeance of the air-gap without taking into account the stator slotting effect.

N_S and N_R are the numbers of stator and rotor slots, P is the number of pole pairs of the motor, ω is the supply

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frequency, s is the slip and k_{C_s} and k_{C_r} are the Carter factors as defined in [4].

In case of rotor eccentricity and in circular whirling motion at the frequency ω_ϵ , the air-gap can be expressed as in [1]:

$$\delta = \delta_0 (1 + \epsilon \cos(\varphi - \omega_\epsilon t + \theta_\epsilon)). \quad (4)$$

A good approximation for the permeance of the air-gap is given by [1]:

$$\frac{1}{\delta_0 (1 + \epsilon \cos(\varphi - \omega_\epsilon t + \theta_\epsilon))} = \Lambda_0 - \Lambda_1 \cos(\varphi - \omega_\epsilon t). \quad (5)$$

The permeance of the air-gap when the rotor is in circular whirling motion and when the slots are taken into account can be rewritten, by substituting (5) in (1), as

$$\Lambda(\varphi, t) = \lambda_s(\varphi, t) \lambda_r(\varphi, t) (\Lambda_0 - \Lambda_1 \cos(\varphi - \omega_\epsilon t)). \quad (6)$$

Assuming a sinusoidal distribution of the magneto-motive force (mmf)

$$F_M = \widehat{F}_M \cos(p\varphi - \omega t), \quad (7)$$

the flux density can be calculated as

$$B(\varphi, t) = F_M(\varphi, t) \Lambda(\varphi, t). \quad (8)$$

By substituting (6) in (8), it is possible to predict the wave numbers and frequencies of the most important harmonics present in the air-gap magnetic flux density of a rotating electrical machine with an eccentric rotor. From the modulating term in (6), it is clear that the eccentric whirling motion leads to eccentricity harmonics associated with every field wave present in the healthy motor. For example, the first stator slot-harmonics and the eccentricity harmonics associated can be written as

$$B_{S1}(\varphi, t) = B_{N_s \pm p} \cos((N_s \pm p)\varphi \mp \omega t) - B_{N_s \pm p \pm 1} \cos((N_s \pm p \pm 1)\varphi \mp (\omega \pm \omega_\epsilon)t) \quad (9)$$

The force, or UMP, is the result of an interaction between two magnetic field waves, the wave numbers of which differ by one [4], [5], so that the total force can be expressed as

$$F = \frac{\pi R l}{4\mu_0} \sum_{\nu} B_{\nu} B_{\nu+1}^* \quad (10)$$

Here ν is the order of the magnetic flux density waves. Most of the terms in this sum are negligible. The non negligible terms are those which contain the fundamental of the flux density or any of the lower orders slot-harmonics. The equation for the force can then be simplified to

$$F = \sum_{\nu} F_{\nu} = \frac{\pi R l}{4\mu_0} \sum_{\nu} (B_{\nu-1} B_{\nu}^* + B_{\nu} B_{\nu+1}^*). \quad (11)$$

Here ν is restricted to the wave numbers of the fundamental and the first slot-harmonics of the flux density, and F_{ν} are the force components corresponding to these flux density waves.

Complex numbers are used in Equations (10) and (11). The real and imaginary parts represent respectively the horizontal and vertical components in a Cartesian coordinate system.

B. Numerical Model Based on the Impulse Method

The impulse method used for the numerical calculations is a standard finite element method for electrical machines, modified in order to be able to simulate the rotor eccentricity in transient mode. At each time-step, the air-gap magnetic flux density is calculated and expressed as spatial Fourier series coefficients. Only the coefficients needed to calculate the force components (11) are stored. For better precision, the force components are not calculated using (11), but with 2D integration of the Maxwell stress tensor over the air-gap [9].

The motor is first started, and once it is in stable operation, an impulse is given to the position of the rotor centre. Then the rotor is returned back to its central position. The frequency response function (FRF) of the force components as function of the whirling frequency is obtained by dividing the Fourier transform (FT) of each force component by the FT of the position of the rotor centre.

III. RESULTS OF THE SIMULATIONS

All results have been obtained for the following machine:

Three-phase 15 kW cage induction motor	
Frequency:	50 Hz
Rated Voltage:	380 V (delta connection)
Number of Pole Pairs:	2
Number of Stator Slots:	36
Number of Rotor Bars:	34
No parallel branches	

Fig. 1 is the graph of the UMP as well as several of the force components as function of the whirling frequency for a 20% eccentricity.

The fundamental force component is the result of an interaction between the magnetic flux density wave with order p and the two eccentricity harmonics with orders $(p-1)$

and $(p+1)$ [2]. The slot-harmonic force components are the results of interactions involving the slot-harmonics of the magnetic flux density and eccentricity harmonics. The force components are calculated analytically from the flux density harmonics obtained from the FEM. The triangles represent points calculated with the whirling method which has been verified experimentally [10].

FIG. 1 HERE

Fig. 2 represents the variation of the UMP and different force components as function of the slip for a static eccentricity. The total UMP, the fundamental force component and the sum of all the slot-harmonic components are plotted.

FIG. 2 HERE

Fig. 3 shows a comparison of different means to reduce the UMP. The first attempt was to close the stator slots using a semi-magnetic material in order to reduce the amplitude of the stator slot harmonics of the magnetic flux density. The second attempt was to close the rotor slots with the same material. The last attempt was to increase the number of rotor slots from 34 to 44 slots. Indeed, the damping of a flux density wave in the air-gap by the rotor bar currents is small when the order of the magnetic field wave is close to the number of rotor slots [5].

FIG. 3 HERE

IV. ANALYSIS AND DISCUSSIONS

The first result is a verification by FEM of the analytical method. Indeed, it is not possible to measure the different force components that form together the UMP, as defined in Chapter 2.1. Instead, if the analytical method is valid, the total UMP should be equal to the sum of all the force components. It is assumed that the higher order components are small and can be neglected. It can be seen on Fig. 1 that the sum of all components is equal to the total UMP. It can also be seen that the slot-harmonic components of the force do not vary much with the whirling frequency at the observed frequencies. Therefore, they are relatively more important at the frequencies where the fundamental part of the force is damped, for example, in case of static eccentricity.

The amplitude of the slot-harmonics depends on the slip [8]. Therefore, the UMP also increases with the slip. It can be seen from Fig. 2 that the slip has a big influence on the amplitude of the UMP in case of static eccentricity.

It can be seen from Fig. 3 that the closing of the stator slots has a positive effect on the reduction of the UMP. Indeed, the stator slot harmonics were reduced and the total UMP decreased at best by 7 %. However the leakage flux at stator windings made the shaft power decrease by 10 %. If we had kept the shaft power constant by increasing the input voltage, the UMP would have increased compared to the original

motor configuration.

After the closing of the rotor slots, the rotor slot harmonics were reduced by 20 %. However, the total UMP increased by 15 % because the damping effect of the rotor bars was attenuated by the presence of the semi-magnetic material. However, at frequencies close to the synchronous frequency, the damping effect is small, and therefore the UMP did not increase. Furthermore, the shaft power was reduced by 25 % because of the leakage flux.

In most motors, the number of rotor slots is close to the number of stator slots, but far from the number of pole pairs. This causes the forces due to the stator slot harmonics to be less damped by the rotor cage than the fundamental force. When the number of rotor bars was increased from 34 to 44, the stator slot harmonics were reduced, and the total UMP was reduced by 12 %. In this test, the shaft power was kept constant.

This paper has provided a numerical method that separates the effects of the slot-harmonics from the fundamental magnetic flux density and has shown that the slot-harmonics are an important source of the UMP. Unfortunately, it appeared to be not easy to reduce the UMP using traditional techniques. Nevertheless, the vibrations caused by the UMP can be reduced by creating a force opposed to the UMP. This is possible in case of active control of vibrations, as for example active magnetic bearings. This paper provides valuable knowledge for the set up and optimization of these methods

V. CONCLUSION

An analytical method that defines the UMP as the sum of force components related to flux density harmonics has been presented and verified numerically by FEM. This definition gives significantly better results than the definition of [2] in the cases where the slot-harmonics of the air-gap magnetic flux density are not negligible. This method was then used to show that the slot harmonics can be an important source of the UMP. The slot harmonic force components also appeared to increase with the slip. The method was then used in different attempts to reduce the UMP by acting on the slot harmonics. The best result was obtained after increasing the number of rotor slots.

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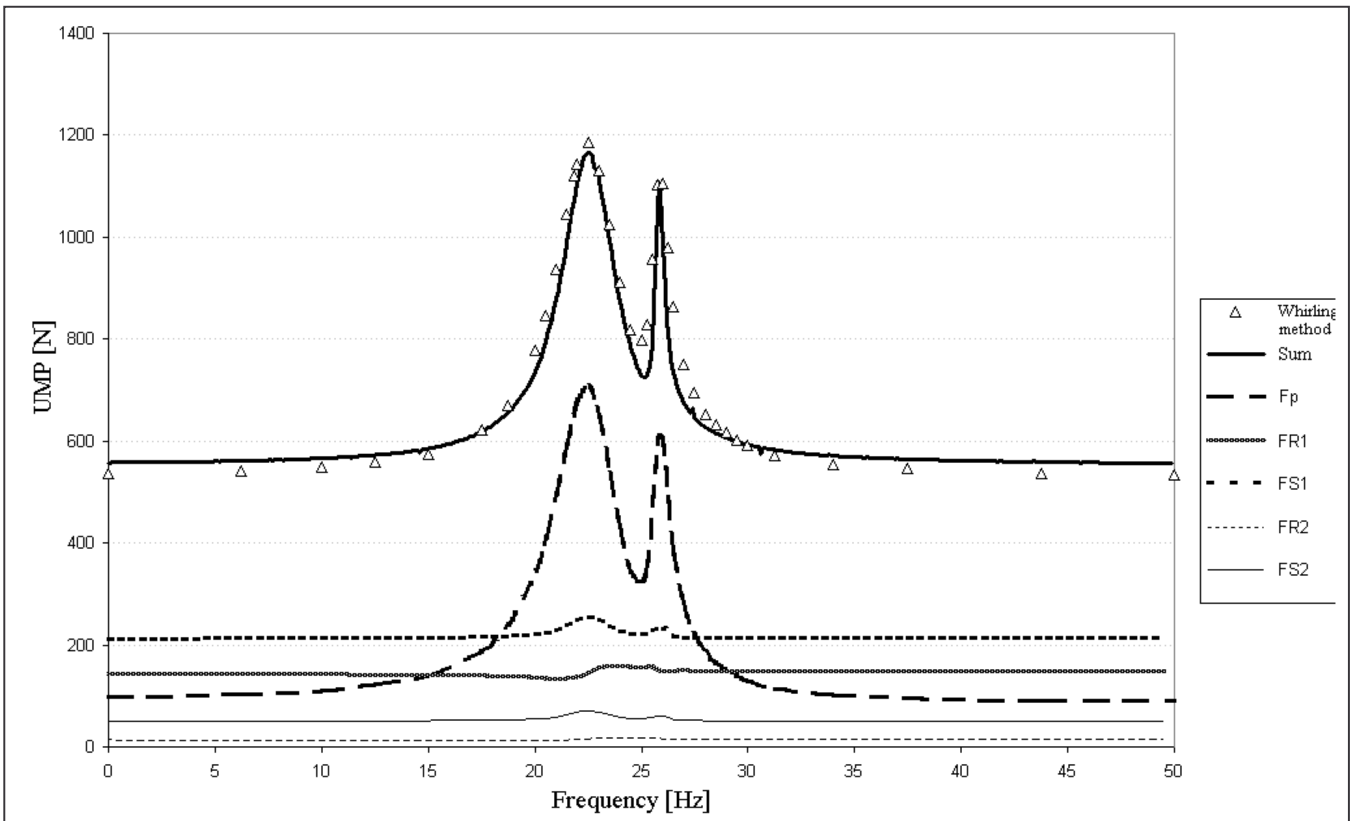


Fig. 1: Total force obtained with the forced whirling method (triangles). Sum of all the force components, up to the fifth order slot-harmonics (continuous line). Several force components are also plotted: fundamental (Fp), first and second rotor slot-harmonics (FR1, FR2), first and second stator slot-harmonics (FS1, FS2).

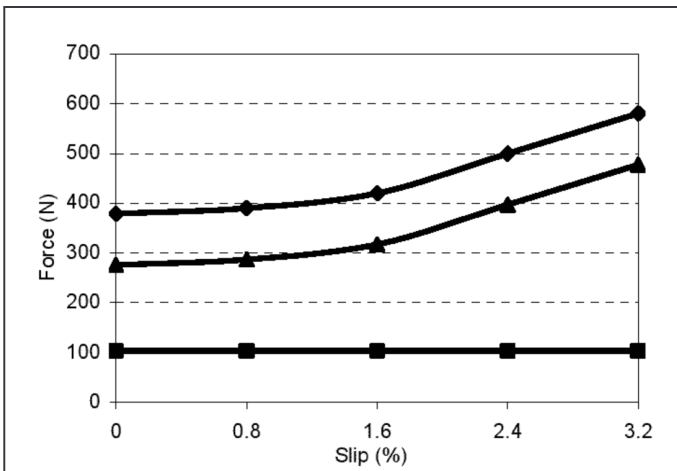


Fig. 2: Variation of the force components with the slip in case of static eccentricity. Are plotted: total UMP (diamond-shaped), slot-harmonics force components (triangles) and fundamental force component (squares).

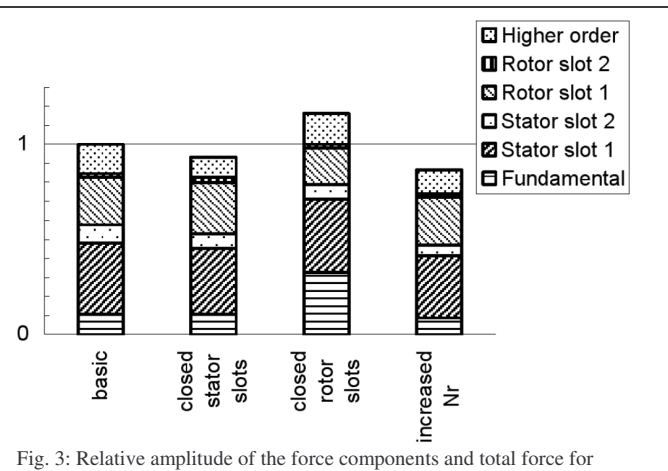


Fig. 3: Relative amplitude of the force components and total force for different motor configurations.